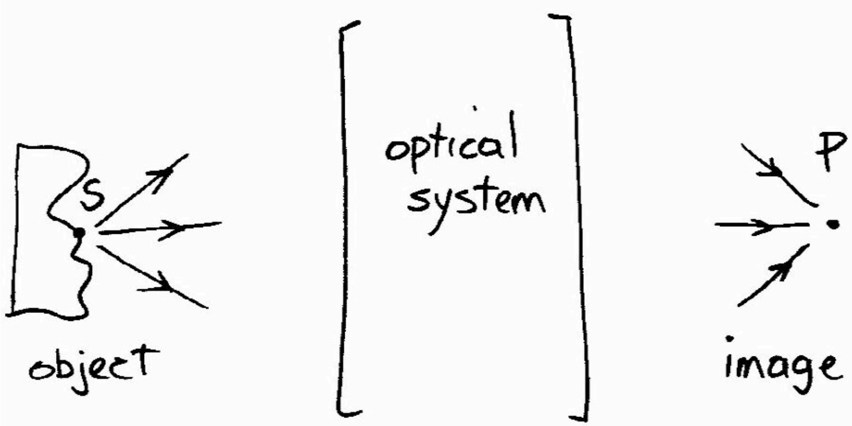
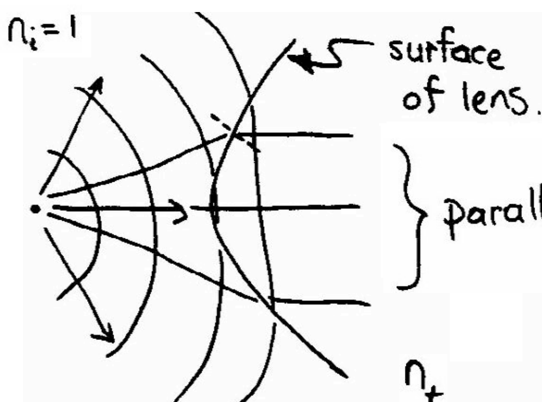
Chap 5 - Geometrical Optics

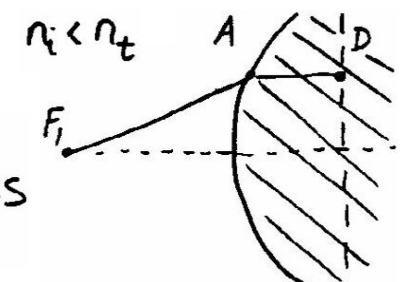


S, P - conjugate points

- diffraction (due to finite acceptance of system)
- abberations (due to imperfect systems)

perfect lenses (aspherical)





$$\phi = \omega\left(\frac{nx}{c} - t\right)$$

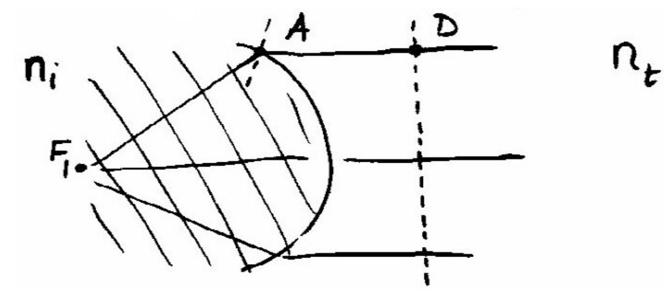
$$\Delta\phi = 0 \Rightarrow n\Delta x = c\Delta t = \text{const.}$$

$$\Rightarrow \quad n_i\,\overline{F_1A} + n_t\,\overline{AD} = \text{const}$$

or $\quad \overline{F_1A} + \left(\frac{n_t}{n_i}\right)\overline{AD} = \text{const.}$

$\Rightarrow$ hyperboloid

alternatively, for $n_i > n_t$

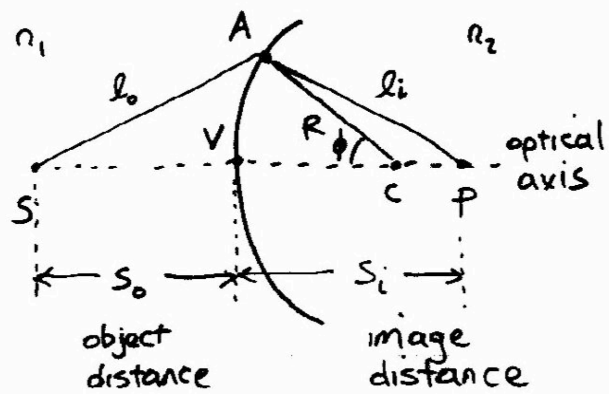


again, $\overline{F_1A} + \left(\frac{n_t}{n_i}\right)\overline{AD} = \text{const.}$

$\Rightarrow$ ellipsoid

- fabrication requires fine computer-controlled milling machines
- easier to make spherical surfaces by mechanical polishing.

Spherical Lenses:



V - vertex (on optic axis)

$OPL = n_1 l_o + n_2 l_i$ use Fermat's Principle

SAC: $l_o = \left[R^2 + (s_o+R)^2 - 2R(s_o+R)\cos\phi\right]^{1/2}$

ACP: $l_i = \left[R^2 + (s_i-R)^2 - 2R(s_i-R)\underbrace{\cos(\pi-\phi)}_{-\cos\phi}\right]^{1/2}$

$$\Rightarrow OPL = n_1\left[R^2 + (s_o+R)^2 - 2R(s_o+R)\cos\phi\right]^{1/2} + n_2\left[R^2+(s_i-R)^2 + 2R(s_i-R)\cos\phi\right]^{1/2}$$

<u>note</u>: all quantities <u>positive</u> (R, s_o, s_i)

$$\frac{d}{d\phi}OPL = +\frac{n_1}{\cancel{2}l_o}\cancel{2}\cancel{R}\left(\underset{R}{\underline{s_o}}+\cancel{R}\right)\sin\phi - \frac{n_2}{\cancel{2}l_i}\cancel{2}\cancel{R}\left(\underset{R}{\underline{s_i}}-\cancel{R}\right)\sin\phi = 0$$

$$\Rightarrow \frac{n_1}{l_o} + \frac{n_2}{l_i} = \frac{1}{R}\left(\frac{n_2 s_i}{l_i} - \frac{n_1 s_o}{l_o}\right)$$

- correct, but complicated

  new $\phi \Rightarrow$ diff. $l_o, l_i \Rightarrow$ point P moves.

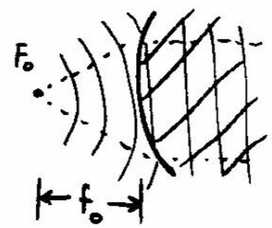
- simplify – assume <u>paraxial</u> <u>rays</u> $(\phi \approx 0)$

  $\Rightarrow l_o \approx s_o$, $l_i \approx s_i$ "1st order, or Gaussian optics"

and $$\boxed{\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2-n_1}{R}}$$

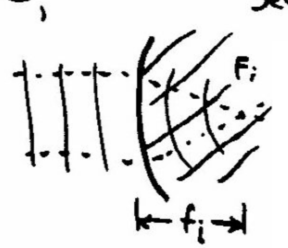
limiting cases: first (object) focal length $s_o \equiv f_o$

$s_i \to \infty$,



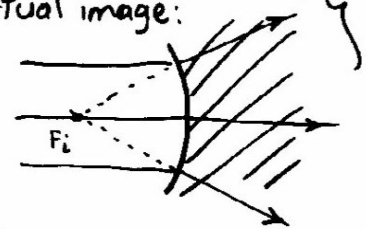
$\frac{n_1}{f_o} = \frac{n_2-n_1}{R}$, $\underline{f_o = R\frac{n_1}{n_2-n_1}}$

$s_o \to \infty$, second (image) focal length $s_i \equiv f_i$



$\frac{n_2}{f_i} = \frac{n_2-n_1}{R}$, $\underline{f_i = R\frac{n_2}{n_2-n_1}}$

virtual image:



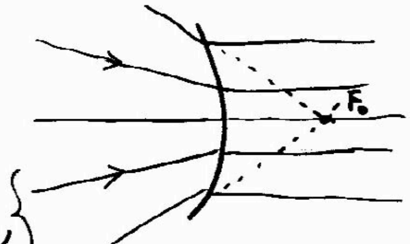
diverging rays

$R<0$

$s_i<0$

$(s_o \to \infty$ here$)$

virtual object:



$R<0$

$s_o<0$

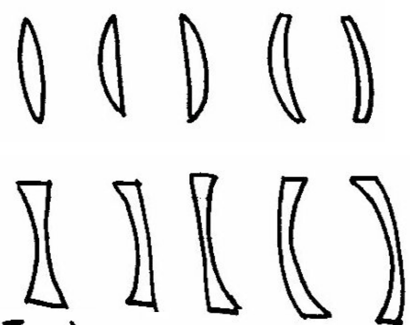
$(s_i \to \infty$ here$)$

converging rays

Thin Lenses:

convex, converging, positive :

concave, diverging, negative :

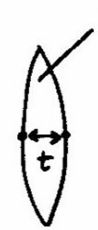


$R_1<0$ $R_2>0$ $R_1>0$ $R_2>0$

$R_2>R_1$

consider refraction from two spherical surfaces:

n_m $\quad R_1>0$ $\quad R_2<0$



1^{st} interface:

$$\frac{n_m}{s_{o1}} + \frac{n_\ell}{s_{i1}} = \frac{n_\ell - n_m}{R_1}$$

2^{nd} interface:

$$\frac{n_\ell}{s_{o2}} + \frac{n_m}{s_{i2}} = \frac{n_m - n_\ell}{R_2}$$

thin lens: $s_{o2} = -s_{i1}$, $\dfrac{n_\ell}{s_{o2}} = -\dfrac{n_\ell}{s_{i1}}$

add: $$\frac{n_m}{s_{o1}} + \frac{n_m}{s_{i2}} = (n_\ell - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

take $n_m \equiv 1$, $s_{o1} \equiv s_o$, $s_{i2} \equiv s_i$ $\Rightarrow$ $$\boxed{\frac{1}{s_o} + \frac{1}{s_i} = (n_\ell - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$ Thin Lens Eqn.

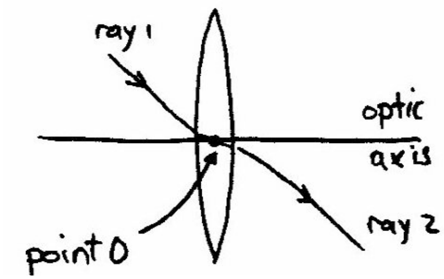
note: $$\lim_{s_i \to \infty} s_o \equiv f_o = \left[(n_\ell - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right]^{-1}$$

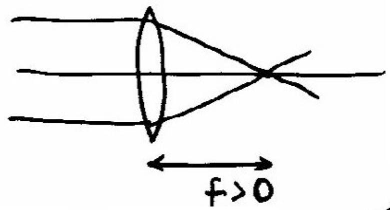
$$\lim_{s_o \to \infty} s_i \equiv f_i = \left[(n_\ell - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right]^{-1}$$

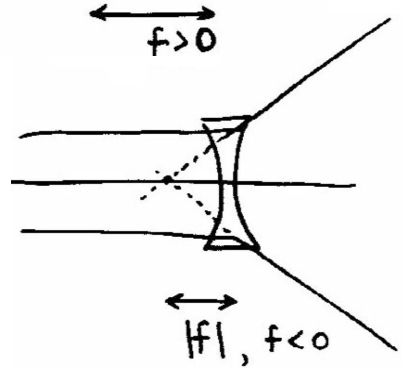
define $$\frac{1}{f} \equiv (n_\ell - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ f is focal length $(f_o = f_i = f)$

then $$\boxed{\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}}$$ Gaussian lens formula.

Ray Diagrams:





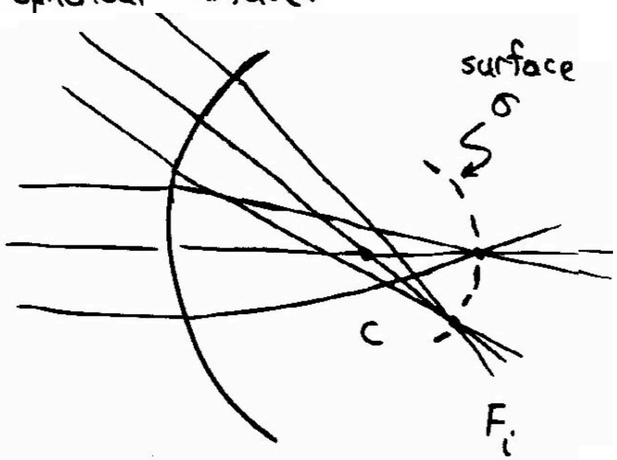


rays 1 and 2 are parallel; colinear for $t \to 0$

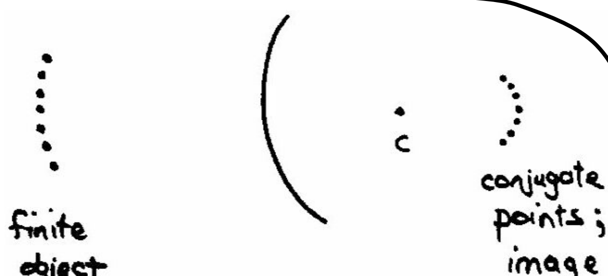
• for all other rays, must use Thin Lens Eqn (but these rays above are sufficient in many cases).

Focussing of Finite-sized objects:

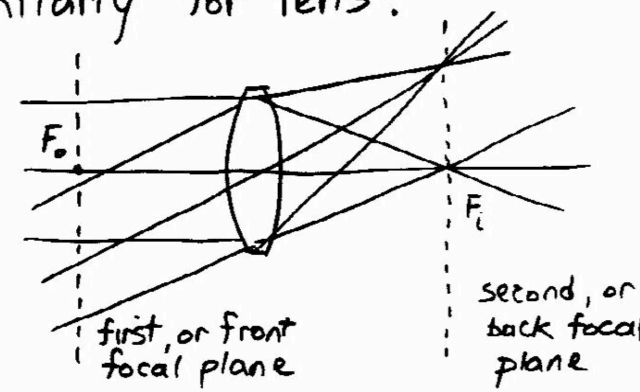
spherical surface:



for paraxial rays, σ is a plane (focal plane)

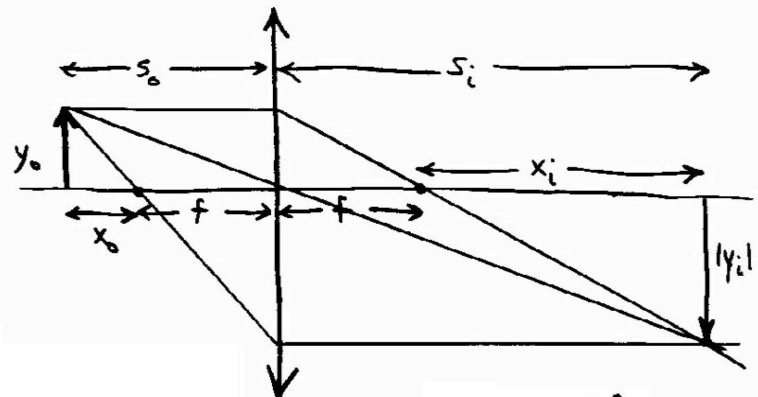


similarly for lens:



(for thin lens, $f_o = f_i \equiv f$)

Magnification (thin lens)



$s_o = x_o + f$

$s_i = x_i + f$

all quantities >0 except $y_i < 0$

ie. inverted image.

see that $\frac{y_o}{|y_i|} = \frac{s_o}{s_i} = \frac{f}{s_i - f}$, $\frac{\cancel{s_i}}{s_o} = \frac{\cancel{s_i}}{f} - \frac{1}{s_i}$

$\Rightarrow \frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$ (thus, construction is correct)

also
$$\frac{|y_i|}{y_o} = \frac{f}{(s_o - f)} = \frac{f}{x_o}$$

and $\frac{y_o}{|y_i|} = \frac{f}{x_i}$ $\Rightarrow$ $\boxed{x_o x_i = f^2}$

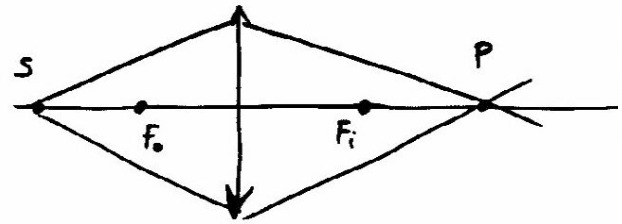
Newton's formula.

magnification (transverse)
$$M_T = \frac{y_i}{y_o} = -\frac{s_i}{s_o} = -\frac{x_i}{f} = -\frac{f}{x_o}$$

Signs:	+	−
s_o	real object	virtual object
s_i	real image	virtual image
f	converging lens	diverging lens
y_o	erect object	inverted object
y_i	erect image	inverted image
M_T	image like object	image opp. object

simple examples:

1)



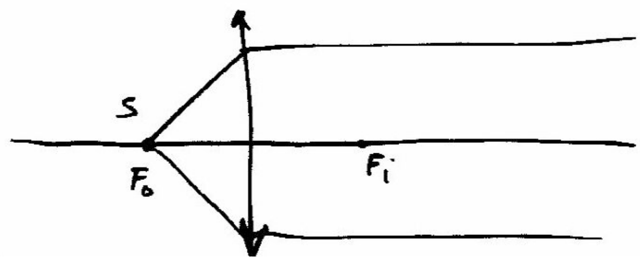
$s_o = 2f$

$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{2f} = \frac{1}{2f}$

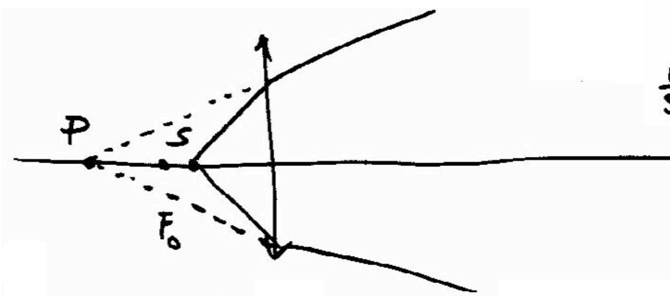
$\Rightarrow s_i = 2f$

$M_T = -1$

2) $s_o = f \Rightarrow s_i = \infty$, $M_T = \infty$



3) $s_o < f \Rightarrow s_i < 0$



eg $s_o = \frac{2}{3}f$

$\frac{1}{s_i} = \frac{1}{f} - \frac{3}{2f} = -\frac{1}{2f}$

$\Rightarrow s_i = -2f$

$M_T = +3$

Also, longitudnal magnification
$$M_L = \frac{dx_i}{dx_o} = -\frac{f^2}{x_o^2} = -M_T^2$$

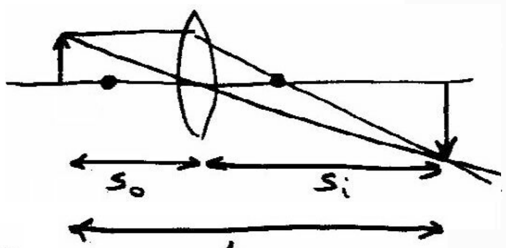
Examples:

A) method of conjugate points:

consider single thin lens: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$

examine image-object distance:

$L \equiv s_o + s_i$



find: $\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{s_o - f}{s_o f}$

$s_i = \frac{s_o f}{s_o - f}$

$L = s_o + \frac{s_o f}{s_o - f} = \frac{s_o^2 - s_o f + s_o f}{s_o - f} = \underline{\underline{\frac{s_o^2}{s_o - f}}}$

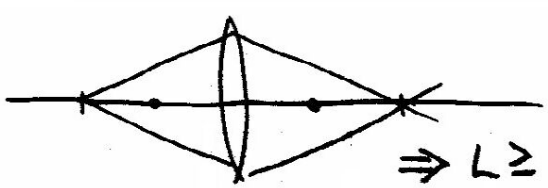
extrema:

$\frac{dL}{ds_o} = \frac{(s_o - f)\cdot 2s_o - s_o^2}{(s_o - f)^2} = \frac{s_o^2 - 2s_o f}{(s_o - f)^2}$

$= 0$ at $s_o = 0$, $\underline{s_o = 2f}$

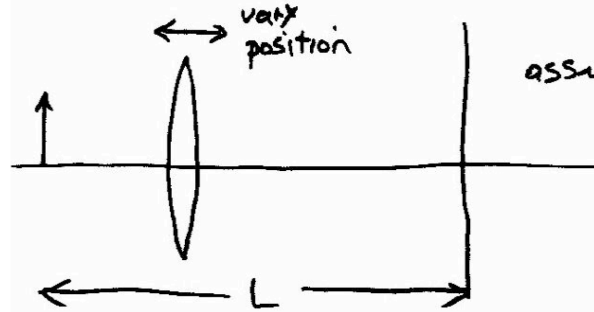
for $s_o = 2f$, $s_i = 2f$

and find that L is a minimum



$\Rightarrow L \geq 4f$ in general.

now, consider arrangement with fixed object and image board, and vary position of lens:



assume $L > 4f$

for which values of s_o (and s_i) is image in focus?

have $L = \frac{s_o^2}{s_o - f}$; solve for s_o,

$s_o^2 - Ls_o + Lf = 0 \quad \Rightarrow \quad s_o = \frac{L \pm \sqrt{L^2 - 4Lf}}{2}$

for $L = 4f$ one root, $s_o = \frac{L}{2} = 2f$

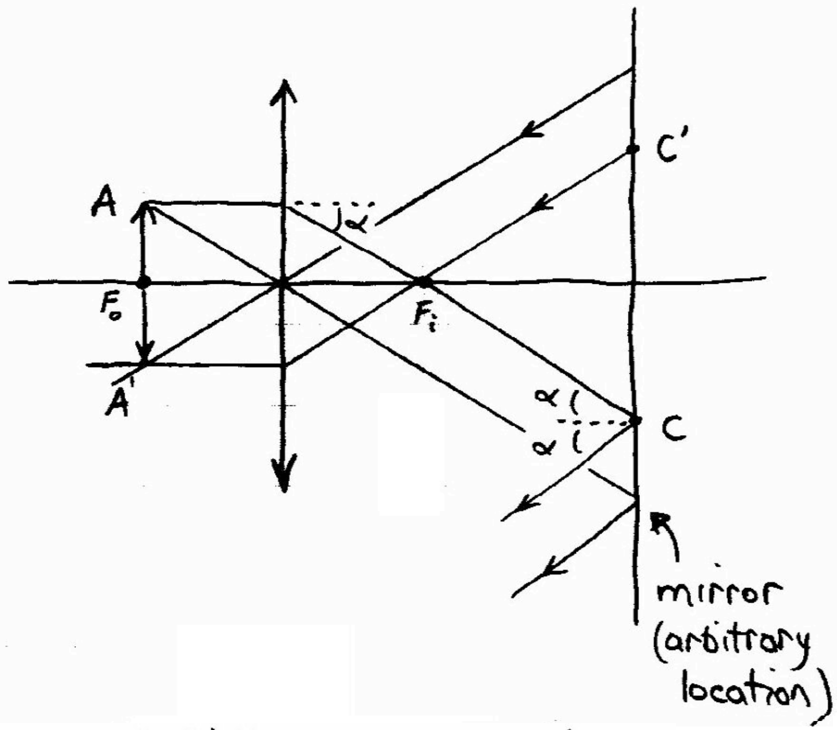
$L > 4f$ two roots, $s_o^{\pm} = \frac{L}{2} \pm \frac{1}{2}\sqrt{L^2 - 4Lf}$

examine separation of roots:

$s_o^+ - s_o^- \equiv d = \sqrt{L^2 - 4Lf} \quad \Rightarrow \quad d^2 = L^2 - 4Lf$

or $\boxed{f = \frac{L^2 - d^2}{4L}}$ can use to measure f.

B) measure f using a mirror:



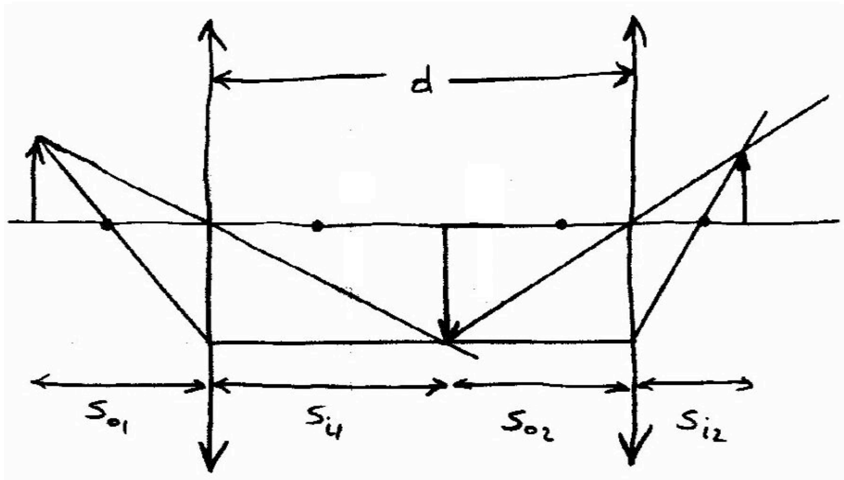
- show that if object is in first focal plane of lens, then image is in same plane, and inverted.
- look at ray from A; reflected from mirror at angle α
- similarly for all rays from A, since it's in focal plane!

- now follow ray from C', located on opposite side of optic axis from C.
- where is that ray (and other parallel rays) focussed?

$\boxed{\text{at } A'}$!

- thus, image is inverted and in same plane as object.
- result is independent of location of mirror (although brightness of image is affected by mirror location).

c) two thin lenses, $d \gg f_1 + f_2$



• expect non-inverted image, with variable M_T.

Work out:

$$\frac{1}{s_{o1}} + \frac{1}{s_{i1}} = \frac{1}{f_1}$$

$$\frac{1}{s_{i1}} = \frac{1}{f_1} - \frac{1}{s_{o1}}$$

$$\Rightarrow s_{i1} = \frac{f_1 s_{o1}}{s_{o1} - f_1}$$

$$\frac{1}{s_{o2}} + \frac{1}{s_{i2}} = \frac{1}{f_2}, \quad s_{o2} = d - s_{i1}$$

$$\frac{1}{s_{i2}} = \frac{1}{f_2} - \frac{1}{d - s_{i1}} = \frac{d - s_{i1} - f_2}{f_2(d - s_{i1})}$$

$$\Rightarrow s_{i2} = \frac{f_2(d - s_{i1})}{d - s_{i1} - f_2}$$

thus,

$$s_{i2} = \frac{f_2\left(d - \frac{f_1 s_{o1}}{s_{o1} - f_1}\right)}{d - \frac{f_1 s_{o1}}{s_{o1} - f_1} - f_2} = \underline{\underline{\frac{f_2 d - \frac{f_1 f_2 s_{o1}}{(s_{o1} - f_1)}}{d - f_2 - \frac{f_1 s_{o1}}{(s_{o1} - f_1)}}}}$$

and, for magnification:

$$M_T = M_{T1} M_{T2} = + \frac{s_{i1}}{s_{o1}} \frac{s_{i2}}{s_{o2}}$$

$$\text{turns out} = \underline{\underline{\frac{f_1 s_{i2}}{d(s_{o1} - f_1) - s_{o1} f_1}}}$$

eg. $f_1 = 30$ cm, $f_2 = 20$ cm, $d = 120$ cm

if $s_{o1} = 50$ cm then $s_{i1} = \frac{30 \cdot 50}{50 - 30} = 75$ cm,

$s_{o2} = 120 - 75 = 45$ cm, $s_{i2} = \frac{20 \cdot 45}{45 - 20} = \underline{\underline{36 \text{ cm}}}$

$M_T = \frac{36}{45} \cdot \frac{75}{50} = \underline{\underline{1.2}}$ agrees with above formula.

front focal length

$s_{i2} \to \infty \Rightarrow s_{o2} \to f_2$

$\Rightarrow s_{i1} \to d - f_2$, and

$$\frac{1}{s_{o1}} = \frac{1}{f_1} - \frac{1}{d - f_2}, \quad \boxed{\text{f.f.l.} = \frac{f_1(d - f_2)}{d - (f_1 + f_2)}}$$

back focal length

$s_{o1} \to \infty \Rightarrow s_{i1} \to f_1$

$\Rightarrow s_{o2} \to d - f_1$

$$\frac{1}{s_{i2}} = \frac{1}{f_2} - \frac{1}{d - f_1}, \quad \boxed{\text{b.f.l.} = \frac{f_2(d - f_1)}{d - (f_1 + f_2)}}$$

how about for other d values?

– same formulas, but results are different ($M_T < 0$ for d small).

for $d \to 0$:

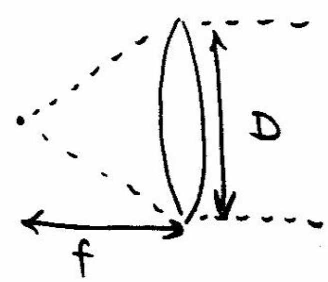
$$b.f.l. = f.f.l. = \frac{f_1 f_2}{f_1 + f_2}$$

looks like single lens, with

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

eg. achromatic doublet; $f_1 > 0$, $f_2 < 0$ to minimize abberations from dispersion

Apertures e.g. diameter of lens restricts light collection



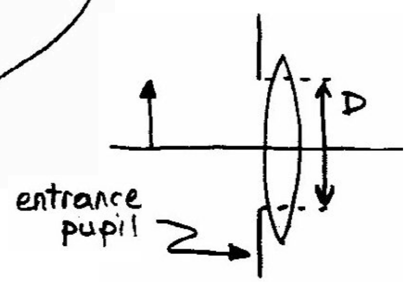
define "f-number"

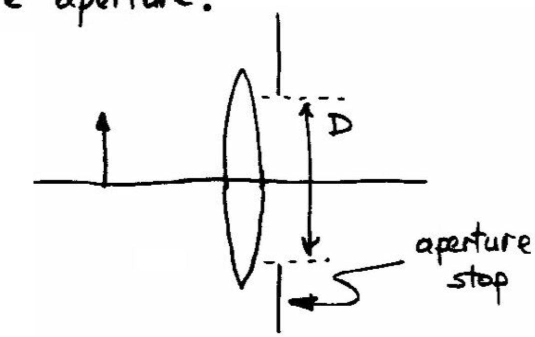
$$f/\# = \frac{f}{D}$$

eg. write as $f/2$ or $f/4$ etc.

smaller $f/\#$ ⇒ more light gathering capability

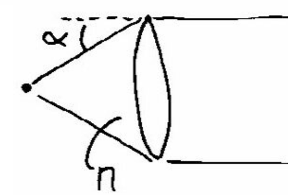
• may use separate aperture:





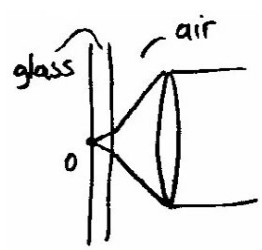
Also, some microscopes quote spec for

numerical aperture $N.A. \equiv n \sin\alpha$

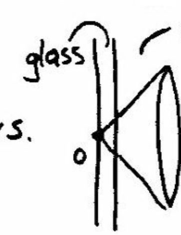


$$\sin\alpha \approx \tan\alpha = \frac{D/2}{f} = \frac{1}{2\cdot(f/\#)}$$

larger n increases light gathering capability;



vs.



can have $N.A. > 1$ for oil immersion lens!